THE POWER SPECTRUM and Applications in Digital EEG

Polysomnography: SIGNALS TO BE ACQUIRED

1) EEG Signal from Scalp Electrodes
2) EMG Signals from Muscles of the Chin
3) ECG Signals from the Heart
4) EOG Signals from the eyes
5) Pulse-oxymetry from blood
6) Breathing Signals close to the nose/mouth
7) Vibration Signals from upper respiratory areas
8) Snoring Signals from a microphone
9) Video Signals from a Video Camera
10) Respiratory effort/body positioning

<u>ELECTROPHYSIOLOGICAL SIGNALS: Challenges of Acquisition and Processing</u>

1.) *Electrical Interferencein EEG and EMG Studies*

Classification of Interference Sources:

a) 50Hz/60Hz magnetic field from power transformers, induction motors etc. (through magnetic coupling)

<u>Basic Solution:</u>

> The avoidance of loops in the pick-up circuits of the EEG or EMG. The magnitude of the interfering voltages is directly proportional to the area enclosed by any such loop.

b) . Magnetic field from staff location systems using inductive loops.

This type of interference is of an intermittent character and may be serious where the inductive loop runs alongside the wall of the examination room and where the transmitted frequencies fall within the pass band of the EEG or EMG/EP.

Most modern staff location systems now use frequencies above 20kHz and should therefore cause minimum interference unless the loop is very close to the EEG or EMG/EP

3. 50Hz/60Hz electrostatic field from unshielded conductors, lamps etc. (through capacitive coupling) This type of interference is possibly the simplest to locate and correct. Any conductor of supply frequency not covered by an earthed metal screen is a potential source of interference. Obvious sources are flexible cables to desk lamps and X-ray viewing boxes, unshielded florescent tubes and filament lamps.

Use metal screens and earth; it is important that all encasing metalwork of electrical equipment be securely bonded electrically to the local circuit earth.

4. HF interference from short-wave diathermy, radio ambulances, television and sound broadcasting

This class of interference is the most difficult to track down to its source and by far the most difficult to *eliminate*.

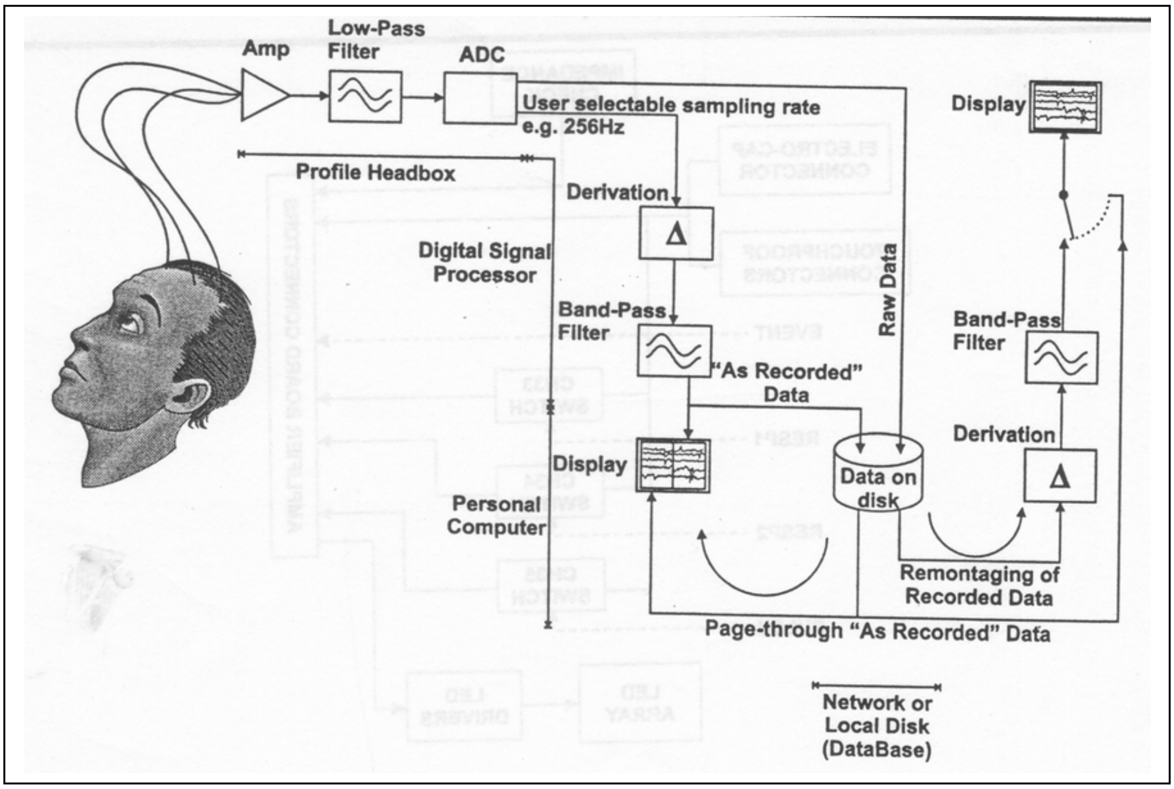
Short-wave diathermy is easily the most frequent source of HF interference, particularly in or near a physical medicine department or operating theatre.

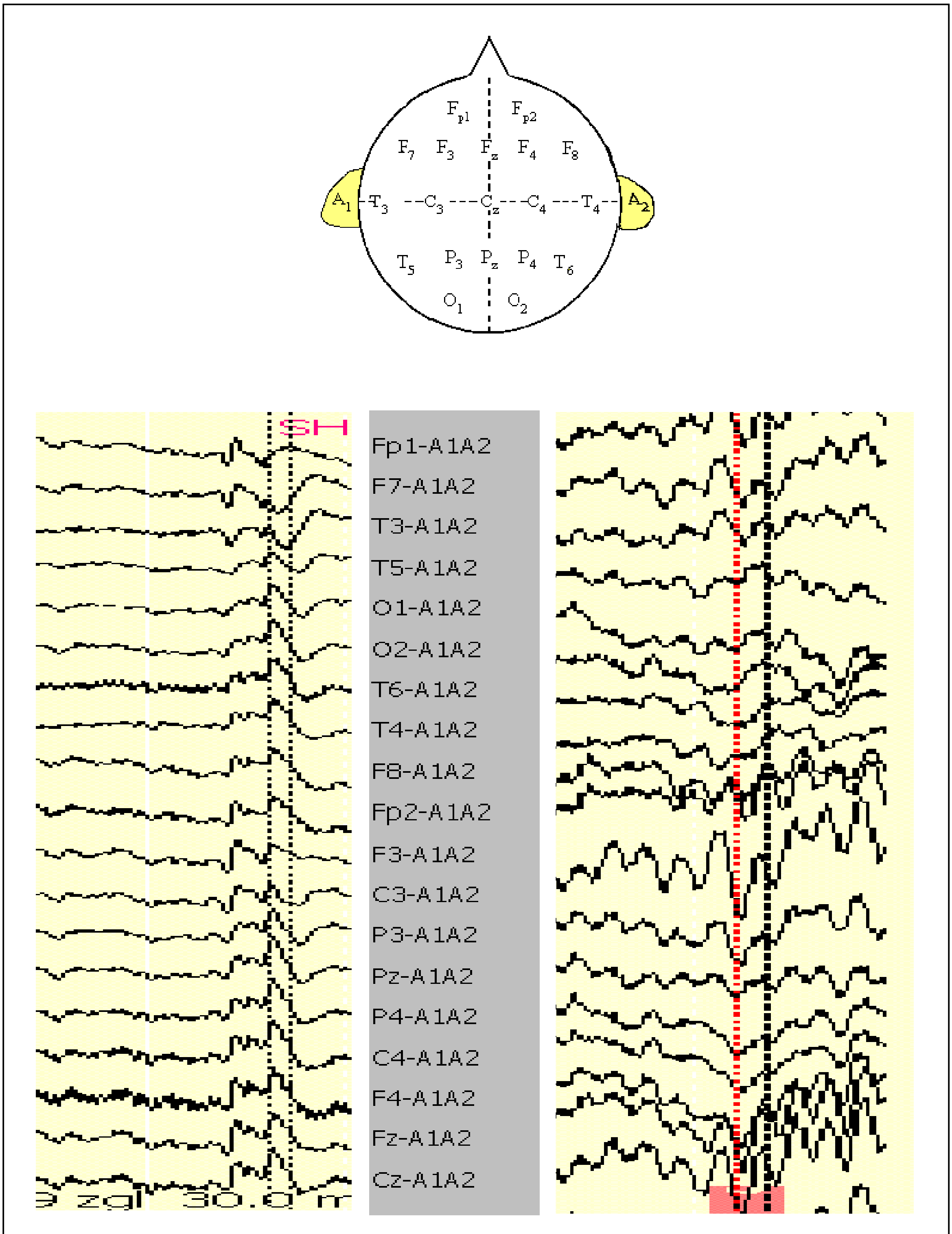
The interference may reach the EEG or EMG/EP via a number of paths falling into two main categories: (1) Conducted and (2) Radiated.

Conducted HF interference is the type, which is found on the mains supply. This kind of interference on modern equipment is eliminated by efficient mains filtering.

Radiated interference is the type transmitted through the air and emitted from either poorly screened electrical equipment or other source such as radio transmitters and hospital paging systems. In severe cases of this kind, a screened enclosure is essential.

A BLOCK DIAGRAM OF A CLINICAL EEG MACHINE

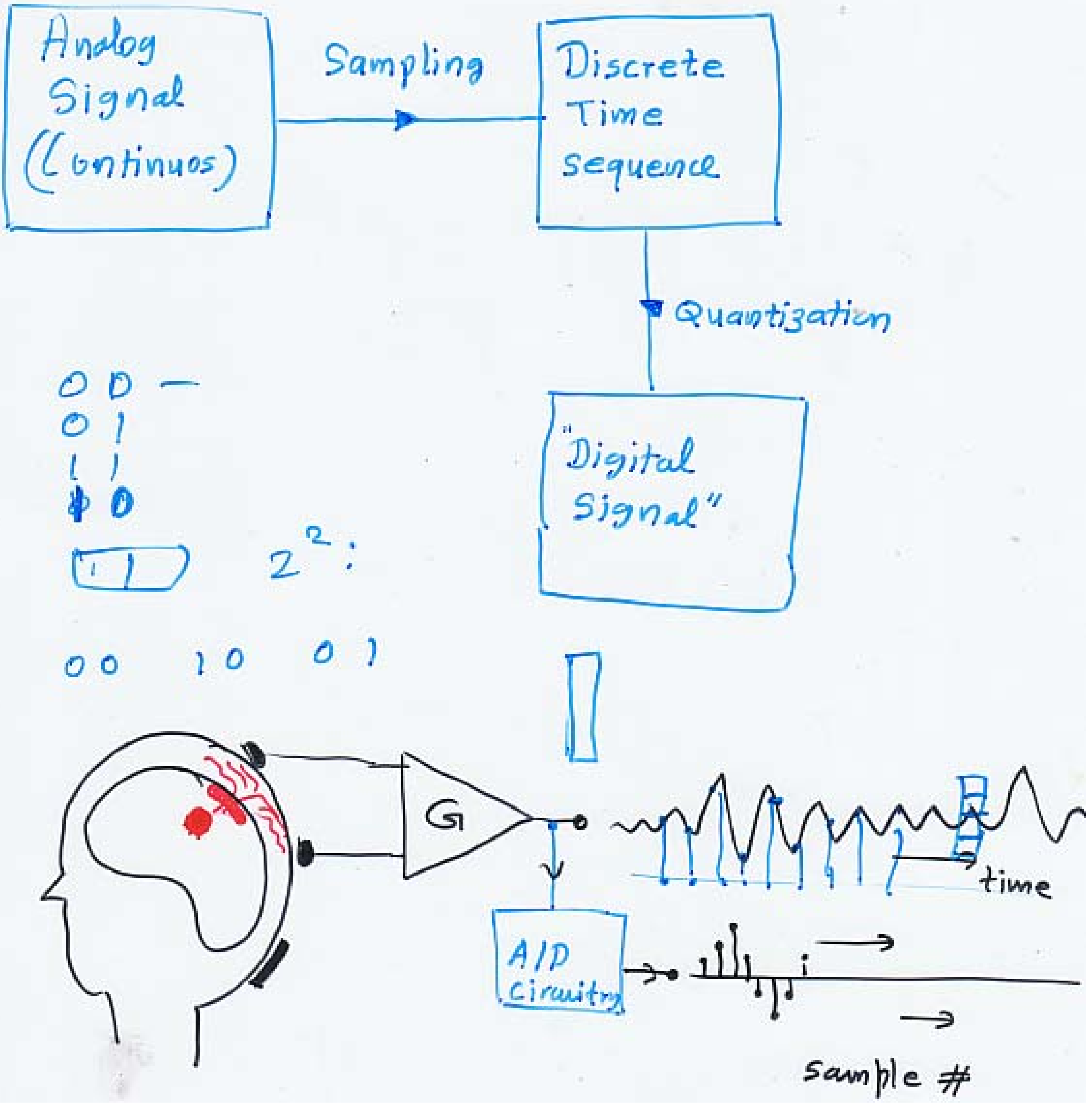



Fp1 Fp2
F7 F3 Fz F4 F8
A1 T3 C3 Cz C4 T4 A2
T5 P3 Pz P4 T6
O1 O2
SH
Fp1-A1A2
F7-A1A2
T3-A1A2
T5-A1A2
O1-A1A2
O2-A1A2
T6-A1A2
T4-A1A2
F8-A1A2
Fp2-A1A2
F3-A1A2
C3-A1A2
P3-A1A2
Pz-A1A2
P4-A1A2
C4-A1A2
F4-A1A2
Fz-A1A2
Cz-A1A2

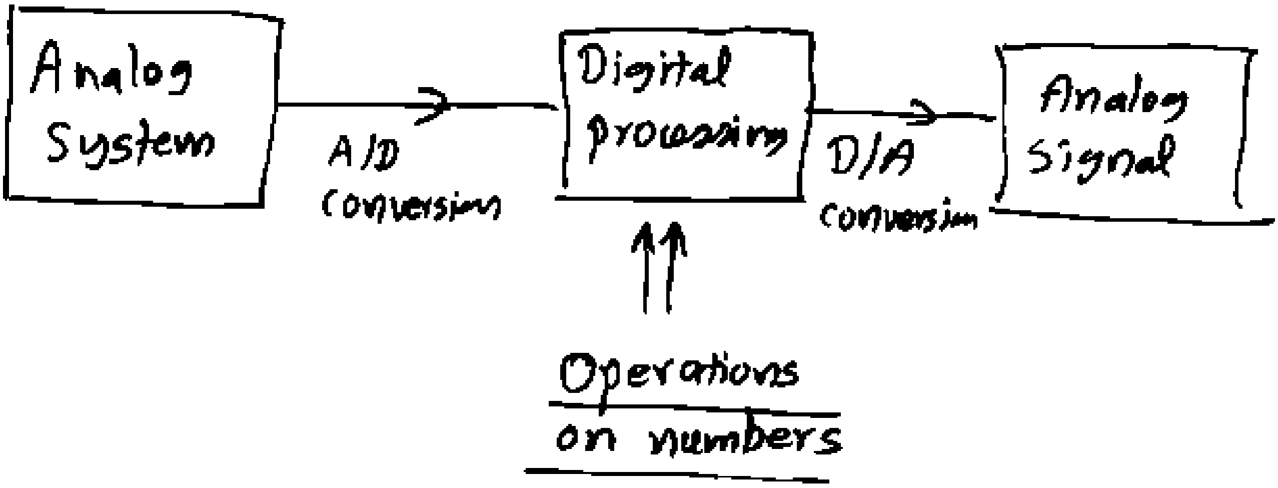
(b)

(c)

Analog-to-Digital Conversion



DSP systems



1) Software based processing
2) Digital hardware based processing
3) "Programmable" hardware (special for DSP)
   - DSP chips
   - Microcontrollers

Transform Domain Descriptions

- Fourier Transform of the continuous Time Signal: (CTFT : Continuous Time Fourier Transform)

$$X_c(\Omega) \triangleq \int_{-\infty}^{\infty} x_c(t) \cdot e^{-j\Omega t} \cdot dt \quad ; \quad \Omega : \text{A real continuous variable.}$$

$$x_c(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\Omega) \cdot e^{j\Omega t} \cdot d\Omega$$

- Z-Transform of the discrete-time Signal:

$$X(z) \triangleq \sum_{n=-\infty}^{\infty} x[n] \cdot z^{-n}$$

$$x[n] = \frac{1}{2\pi j} \oint X(z) \cdot z^{n-1} \cdot dz$$

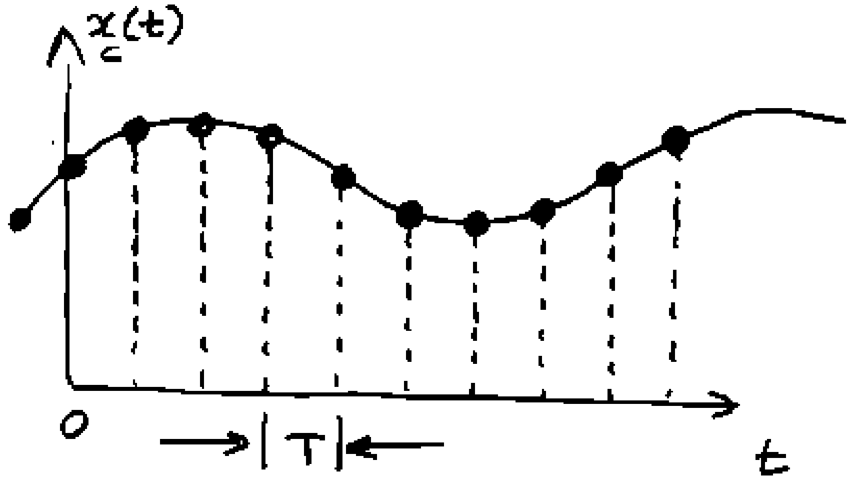
- DTFT : Discrete-Time Fourier Transform
Fourier Transform of a discrete time Seq:

$$X(z)\Big|_{z=e^{j\omega}} \triangleq X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] \cdot e^{-j\omega n}$$

$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) \cdot e^{j\omega n} \cdot d\omega$$

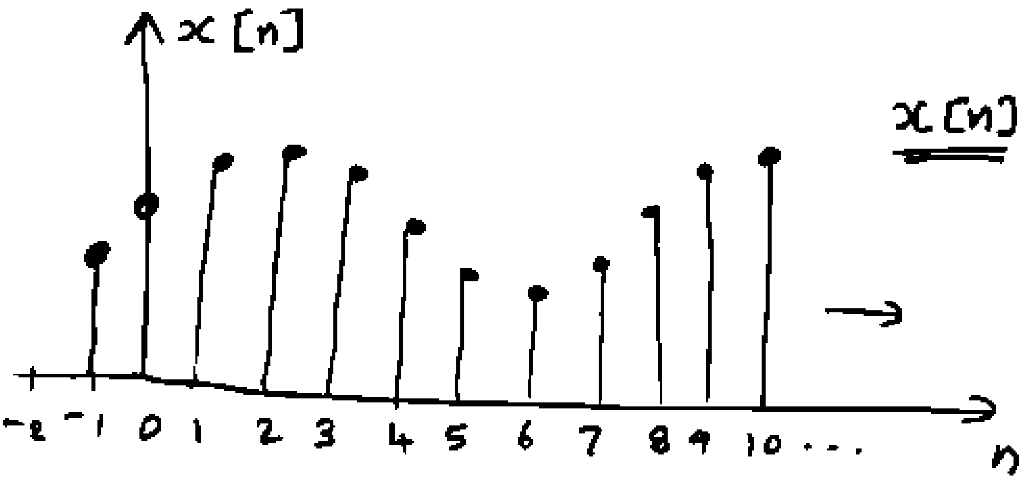
ω = real, continuous variable

The Sampling Process



$x_c(t)$
Continuous-Time Signal

- T : sampling interval (s)
- $\frac{1}{T} \triangleq f_s$: sampling frequency (samples/s)



$x[n]$: Discrete-Time Sequence
: Time Series

- n = Sample number (unitless/dimensionless)

$$x[n] = x_c(nT), \quad \text{for all } n.$$

In the digital signal processing world, we play with $x[n]$ to modify $x_c(t)$ the way we want!

The sampling Theorem

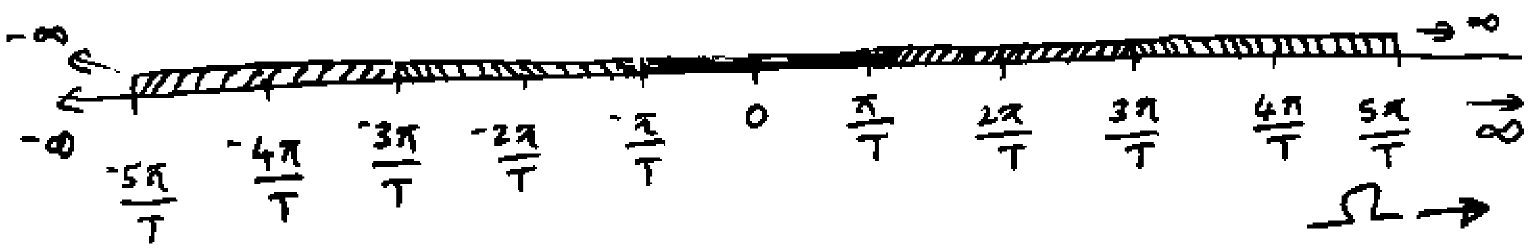
Fourier Transform of $x_c(t)$:

$$X_c(\Omega) = \int_{-\infty}^{\infty} x_c(t)\cdot e^{-j\Omega t}\cdot dt$$

$$x_c(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X_c(\Omega)\cdot e^{j\Omega t}\cdot d\Omega \qquad (-\infty<\Omega<\infty)$$

$$x[n] = x_c(nT) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X_c(\Omega)\cdot e^{j\Omega nT}\cdot d\Omega\ , \quad -\infty<\Omega<\infty$$

Partition the Ω-axis, $-\infty<\Omega<\infty$.



$$(-\infty,\infty) \equiv \Big\{ \left[-\tfrac{\pi}{T},\tfrac{\pi}{T}\right], \left[\tfrac{\pi}{T},\tfrac{2\pi}{T}\right], \cdots\cdots$$

$$\left[-\tfrac{3\pi}{T},-\tfrac{\pi}{T}\right], \left[-\tfrac{5\pi}{T},-\tfrac{3\pi}{T}\right]\cdots\cdots \Big\}$$

$$\text{“}\equiv\text{”} \ \text{“}\sum_{k=-\infty}^{\infty}\text{”} \left[(2k-1)\tfrac{\pi}{T},\ (2k+1)\tfrac{\pi}{T}\right]$$

$$x[n] = x_c(nT) = \frac{1}{2\pi}\left\{ \cdots + \int_{-\frac{\pi}{T}}^{\pi/T} X_c(\Omega)\cdot e^{j\Omega nT}\, d\Omega \right.$$

$$+ \int_{\pi/T}^{3\pi/T} X_c(\Omega)\cdot e^{j\Omega nT}\, d\Omega$$

$$+$$

$$\left. \vdots \right\}$$

$$\boxed{x[n] = \frac{1}{2\pi}\sum_{k=-\infty}^{\infty} \int_{(2k-1)\frac{\pi}{T}}^{(2k+1)\frac{\pi}{T}} X_c(\Omega)\cdot e^{j\Omega nT}\cdot d\Omega}$$

<u>change the variable ω to:</u>

$$\beta = \Omega - \frac{2\pi k}{T}$$

$$x[n] = \frac{1}{2\pi}\sum_{k=-\infty}^{\infty} \int_{-\pi/T}^{\pi/T} X_c\left(\beta + \frac{2\pi k}{T}\right)\cdot e^{j\left(\beta + \frac{2\pi k}{T}\right)nT}\cdot d\beta$$

$$e^{jnT(2\pi k/T)} = 1$$

$$x[n] = \frac{1}{2\pi}\sum_{k=-\infty}^{\infty} \int_{-\frac{\pi}{T}}^{\frac{\pi}{T}} X_c\left(\beta + \frac{2\pi k}{T}\right)\cdot e^{j\beta nT}\cdot d\beta$$

$$x[n] = \frac{1}{2\pi} \int_{-\frac{\pi}{T}}^{\frac{\pi}{T}} \left[\sum_{k=-\infty}^{\infty} X_c\left(\beta + \frac{2\pi k}{T}\right) \right] \cdot e^{j\beta n T} \cdot d\beta$$

Let $\omega = \beta T$ (Another change of variables)

$$x[n] = \frac{1}{2\pi} \cdot \int_{-\pi}^{\pi} \left[\frac{1}{T} \cdot \sum_{k=-\infty}^{\infty} X_c\left(\frac{\omega + 2\pi k}{T} \right) \right] e^{j\omega n} \cdot d\omega$$

Note

$$X(e^{j\omega}) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X_c\left(\frac{\omega + 2\pi k}{T} \right)$$

DTFT of the sequence $x[n]$ is

$$X(e^{j\omega}) \triangleq \sum_{n=-\infty}^{\infty} x[n] \cdot e^{-j\omega n}$$

The relationship between the CTFT and DTFT

$$\underbrace{\sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}}_{\text{DTFT}} = X(e^{j\omega}) = \frac{1}{T} \sum_{k=-\infty}^{\infty} \underbrace{X_c\left(\frac{\omega + 2\pi k}{T} \right)}_{\text{CTFT}}$$

Σ scaled/shifted

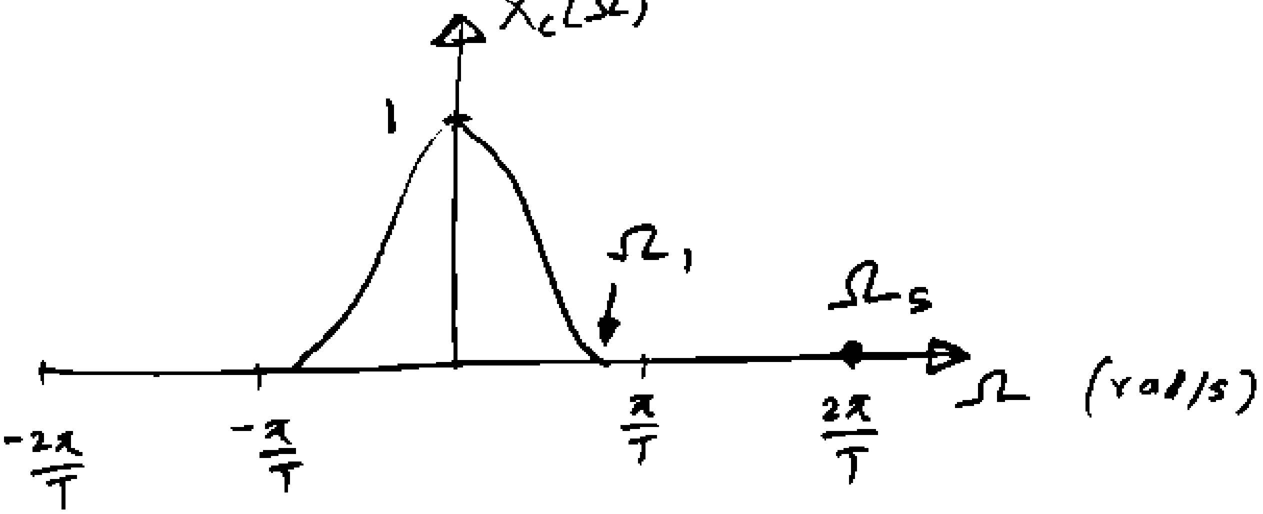
- Sampling Interval $= T$ seconds
- Sampling Frequency $= \frac{1}{T}$ samples/second.

$= f_s$ ("Hz")

$= 2\pi f_s$ in "rad/s"

$= \Omega_s$

$$\Omega_s = \frac{2\pi}{T}$$



- DTFT is a summation of an infinite number of replications of the CTFT:

  (i) Each replication is scaled by $\frac{1}{T}$ and shifted along the frequency axis by multiples of Ω_s.

$$X(e^{j\omega}) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_c\Big(\underbrace{\frac{\omega}{T}}_{\Omega} + k\cdot\Omega_s\Big)$$

- The scaling factor is given by:

$$\omega = \Omega T \quad \Leftarrow \quad \frac{\omega}{T} = \Omega$$

$$\omega = \Omega \cdot \frac{2\pi}{2\pi} \cdot \frac{1}{f_s} = \frac{\Omega}{\Omega_s}\cdot(2\pi)$$

When $\Omega = \Omega_s$

$$\omega = 2\pi$$

lationship

$$X_C(\Omega) = T\,X(e^{j\Omega T}), \qquad -\frac{\Omega_S}{2} \le \Omega \le \frac{\Omega_S}{2}. \tag{7.37}$$

That is, when the CTFT of a signal $x(t)$ is zero for all frequencies above some $f = f_1$ (or $\Omega = \Omega_1$), then the CTFT of $x(t)$ is recoverable from the DTFT of its samples, $x[n]$, as long as the sampling frequency is higher than $2f_1$ (or $2\Omega_1$). This condition is

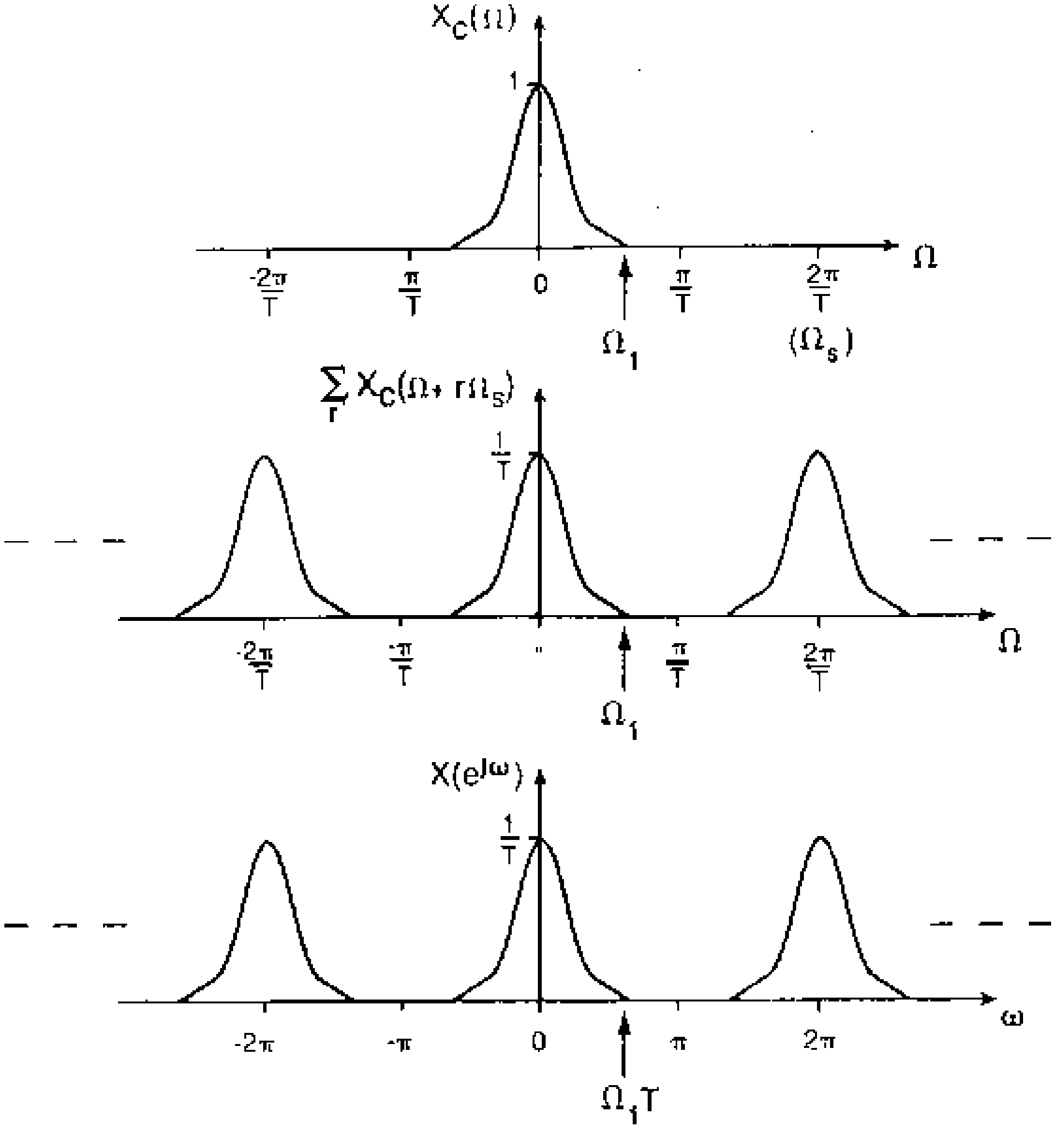


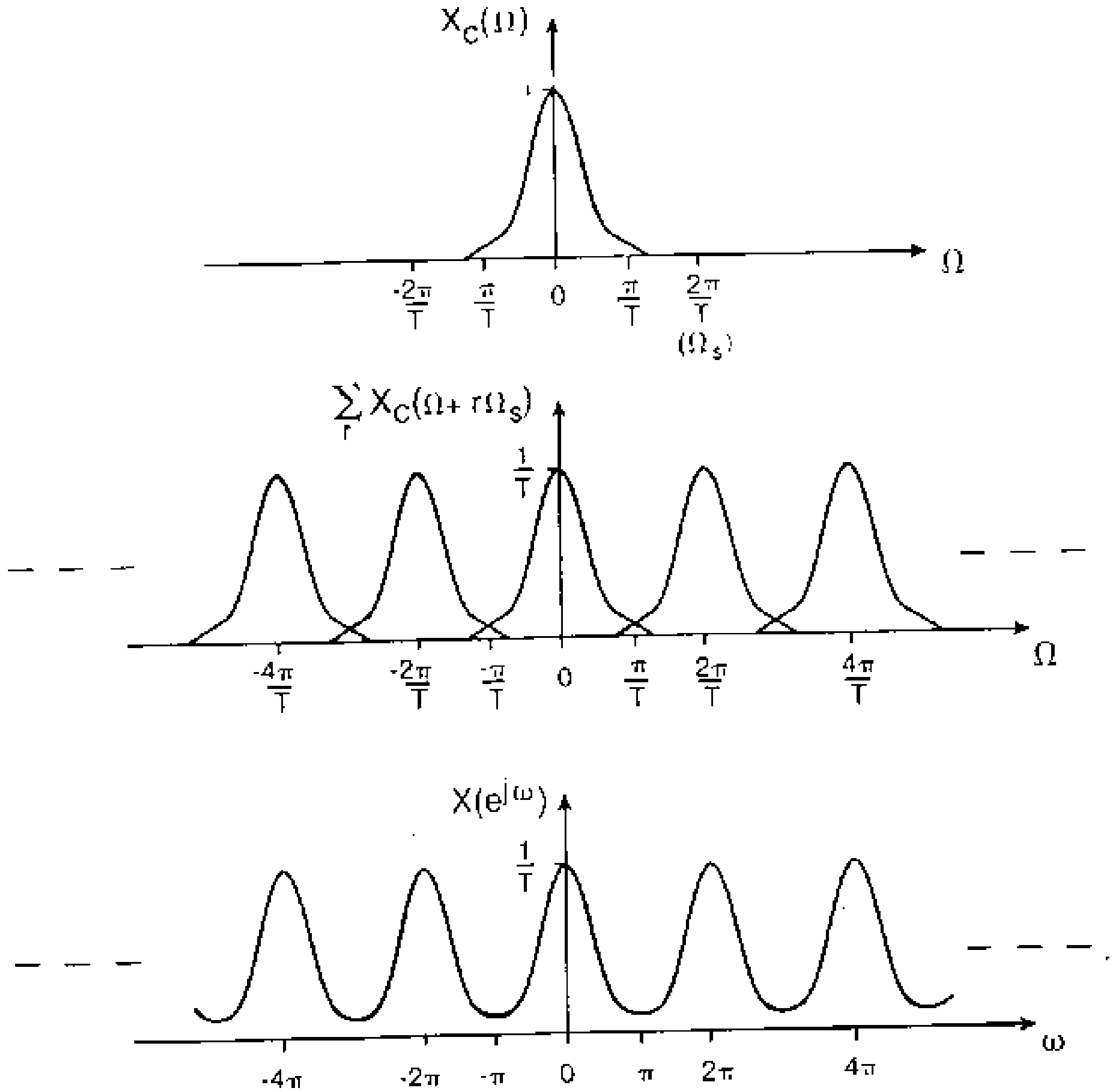
FIGURE 7.13. The effect of sampling on the Fourier spectrum. Top: Fourier transform of a band-limited CT signal, $x(t)$. Middle: Replication of the CTFT, with replicates centered at integer multiples of the sampling frequency, that is implied by the sampling theorem. Bottom: DTFT of the sampled signal, $x[n]$.

In order to recover $X_c(\Omega)$ completely from $X(e^{j\omega})$:

$$X_c(\Omega) = 0 \quad \text{for } |\Omega| \geq \frac{\pi}{T} = \frac{\Omega_s}{2}$$

- "Sampling Theorem"
- [illegible]
- Nyquist Frequency $= 2\Omega_1$ rad/s
  $= 2f_1$ Hz

  where Ω_1 (f_1) is the highest f. component in the signal.



URE 7.14. Same as Fig. 7.13 except that the sampling frequency is less than the Nyquis uency.

Sampling of Random Signals

Continuous Time Random Signal $\xrightarrow{\text{sampling}}$ Discrete-Time random signal.

- What happens to the mean, variance and the co-variance of the Signal?

$$E\{x[n]\} = E\{x_c(nT)\} = \mu_x = E\{x_c(n)\}$$

$$E\{(x[n]-\mu_x)^2\} = E\{(x_c(nT)-\mu_x)^2\} = \sigma_x^2$$

Co-variance

$$\begin{aligned}
\gamma_x[m] &= E\{(x[n+m]-\mu_x)(x[n]-\mu_x)\} \\
&= E\{(x_c(nT+mT)-\mu_x)(x_c(nT)-\mu_x)\} \\
&= \gamma_{x_c}(mT)
\end{aligned}$$

1). Sampling of a stationary random signal $\longrightarrow$ discrete time stationary random signal.

2). The co-variance sequence of the sampled sequence the same as the co-variance of the continuous signal sampled at the same sampling interval.

Power Spectral Density

(and the Power Spectrum)

⊛ Power Spectral Density $S_{xx}(z)$ of $x[n]$:

$$S_{xx}(z) = \sum_{k=-\infty}^{\infty} R_{xx}(k)\, z^{-k}$$

Wiener - Khintchin Theorem.

$$E\{x[n]\, x[n+k]\} = R_{xx}(k)$$

⊛ Power Spectrum

$$S_{xx}(\omega) = \sum_{k=-\infty}^{\infty} R_{xx}(k)\cdot e^{-j\omega k}$$

inverse Fourier

inverse z

$$R_{xx}[k] = \frac{1}{2\pi}\int_{-\pi}^{\pi} S_{xx}(\omega)\cdot e^{j\omega k}\, d\omega = \oint \frac{S_{xx}(z)\cdot z^{k}\, dz}{2\pi j z}$$

⊛ $R_{xx}[k] = E\{x[n]\, x[n+k]\}$

$R_{xx}[0] = E\{x[n]\cdot x[n]\} = E\{x^2[n]\}$

= Total value of Power Expected in $x(n)$.

$$R_{xx}(0) = \text{Total Power} = \int_{-\pi}^{\pi} S_{xx}(\omega) \cdot \frac{d\omega}{2\pi}$$

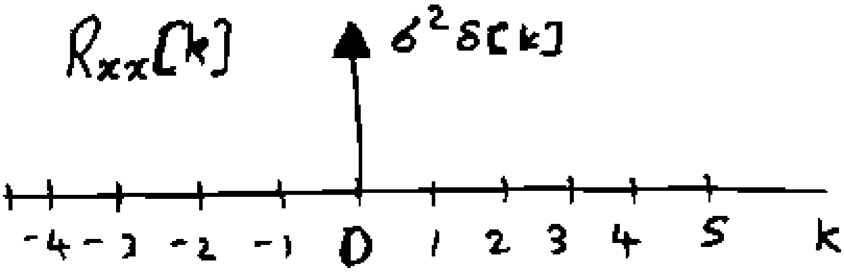
<u>Note:</u> $\omega = 2\pi f$

rad/s Hz

$S_{xx}(\omega)$: Measure of Power per unit frequency interval.

* If $x[n]$ is a white-noise process

$$R_{xx}[k] = E\{x[n]\,x[n+k]\} = \sigma^2 \cdot \delta[k]$$



Power Spectral Density:

$$PSD = \mathcal{Z}\left[R_{xx}[k]\right] = \sigma^2$$

Power Spectrum :

$$S_{xx}(\omega) = \mathcal{F}\left\{R_{xx}[k]\right] = \sigma^2 \quad \text{for all } \omega.$$